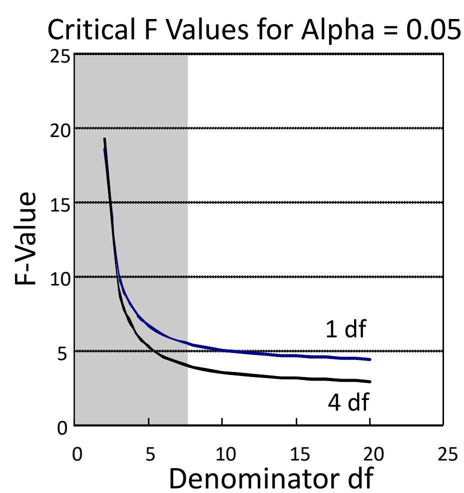
Expected Mean Squares
Pooling Error Terms

Pooling Philosophies:
- Never
- Always – pool at α = 0.05
- Sometimes – pool at α = 0.25

$$MS_p = \frac{SS_1 + SS_2}{df_1 + df_2}$$

Expected Mean Squares
Pooling Error Terms

Why pool error terms?
- df for error terms are sometimes too low for a robust F test (< 6)
- Common when e.u.s are expensive:
  - Growth chambers
  - Greenhouses
  - Animals
- Pooling random terms that are not different increases df for error



Pooling Example – Sorghum Experiment

Source	DF	SS	MS	F Value	Pr > F
Management	2	0.0139	0.0070	0.81	0.447
N Rate	2	0.0643	0.0321	3.75	0.027
M*N	4	0.0837	0.0209	2.44	0.052
Harvest	4	15.1831	3.7958	443.01	<.0001
M*H	8	0.0498	0.0062	0.73	0.667
N*H	8	0.0741	0.0093	1.08	0.383
M*N*H	16	0.1538	0.0096	1.12	0.348
Error	**90**	0.7711	0.0086		

Source	DF	SS	MS	F Value	Pr > F
Management	2	0.0139	0.0070	0.81	0.447
N Rate	2	0.0643	0.0321	3.74	0.027
M*N	4	0.0837	0.0209	2.43	0.051
Harvest	4	15.1831	3.7958	441.47	<.0001
Error	**122**	1.0490	0.0086		

Expected Mean Squares

Estimating Variance Components

$$\sigma_n^2 = \frac{MS_n - MS_d}{c}$$

Where c = coefficient from variance term in EMS.

Estimating Variance Components
Example

Source	EMS
A_i	$\sigma^2 + r\sigma^2_{AB} + br\sigma^2_A$
B_j	$\sigma^2 + r\sigma^2_{AB} + ar\sigma^2_B$
AB_{ij}	$\sigma^2 + r\sigma^2_{AB}$
$\varepsilon_{(ij)k}$	σ^2

σ^2_A $= [(\sigma^2 + r\sigma^2_{AB} + br\sigma^2_A) - (\sigma^2 + r\sigma^2_{AB})] / br$

$= [MS(A) - MS(AB)] / br$

σ^2_B $= [(\sigma^2 + r\sigma^2_{AB} + ar\sigma^2_B) - (\sigma^2 + r\sigma^2_{AB})] / ar$

$= [MS(B) - MS(AB)]/ ar$

σ^2_{AB} $= [(\sigma^2 + r\sigma^2_{AB}) - (\sigma^2)] / r$

$= [MS(AB) - MS(E)] / r$

Estimating Variance Components
Lorenzen and Anderson, Example 3, p. 95

Source	MS	EMS
Field	124.82	$\sigma^2 + 25\sigma^2_F$
Fertilizer Rate	711.77	$\sigma^2 + 5\sigma^2_{FR} + 10\Phi(R)$
Field x Rate	69.87	$\sigma^2 + 5\sigma^2_{FR}$
Error	16.29	σ^2

$$\hat{\sigma}^2_F = \frac{MS(F) - MS(E)}{25} = \frac{124.82 - 16.29}{25} = 4.34$$

$$\hat{\sigma}^2_{FR} = \frac{MS(FR) - MS(E)}{5} = \frac{69.87 - 16.29}{5} = 10.72$$

$$\hat{\sigma}^2_E = MS(E) = 16.29$$

Expected Mean Squares
Lorenzen and Anderson, Problem 3.20

Model:

$Y_{ijk} = \mu + A_i + B_j + AB_{ij} + C_k + AC_{ik} + BC_{jk} + ABC_{ijk}$

Factors:

A	3	Random
B	4	Random
C	2	Random

Expected Mean Squares
Lorenzen and Anderson, Problem 3.20

Source	3 R i	4 R j	2 R k	Expected Mean Square
A_i				
B_j				
AB_{ij}				
C_k				
AC_{ik}				
BC_{jk}				
ABC_{ijk}				

Expected Mean Squares
Lorenzen and Anderson, Problem 3.20

Source	3 R i	4 R j	2 R k	Expected Mean Square
A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_A$
B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_B$
AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_C$
AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

<u>Direct Tests</u>:
All 2-way interactions are tested by MS(ABC)

Source	3 R i	4 R j	2 R k	Expected Mean Square
A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_A$
B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_B$
AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_C$
AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

Approximate Tests:
A = MS(A) / MS(AC) + MS(AB) - MS(ABC)
B = MS(B) / MS(BC) + MS(AB) - MS(ABC)
C = MS(C) / MS(BC) + MS(AC) - MS(ABC)

	Source	3 R i	4 R j	2 R k	Expected Mean Square
	A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_{A}$
	B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_{B}$
+	AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
	C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_{C}$
+	AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
	BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
-	ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

Approximate Tests:
A = MS(A) / MS(AC) + MS(AB) - MS(ABC)
B = MS(B) / MS(BC) + MS(AB) - MS(ABC)
C = MS(C) / MS(BC) + MS(AC) - MS(ABC)

	Source	3 R i	4 R j	2 R k	Expected Mean Square
	A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_{A}$
	B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_{B}$
+	AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
	C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_{C}$
	AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
+	BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
-	ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

Approximate Tests:
A = MS(A) / MS(AC) + MS(AB) - MS(ABC)
B = MS(B) / MS(BC) + MS(AB) - MS(ABC)
C = MS(C) / MS(BC) + MS(AC) - MS(ABC)

	Source	3 R i	4 R j	2 R k	Expected Mean Square
	A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_{A}$
	B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_{B}$
	AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
	C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_{C}$
+	AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
+	BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
-	ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

No Test:
ABC

Source	3 R i	4 R j	2 R k	Expected Mean Square
A_i	1	4	2	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC} + 2\sigma^2_{AB} + 8\sigma^2_{A}$
B_j	3	1	2	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 2\sigma^2_{AB} + 6\sigma^2_{B}$
AB_{ij}	1	1	2	$\sigma^2 + \sigma^2_{ABC} + 2\sigma^2_{AB}$
C_k	3	4	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC} + 4\sigma^2_{AC} + 12\sigma^2_{C}$
AC_{ik}	1	4	1	$\sigma^2 + \sigma^2_{ABC} + 4\sigma^2_{AC}$
BC_{jk}	3	1	1	$\sigma^2 + \sigma^2_{ABC} + 3\sigma^2_{BC}$
ABC_{ijk}	1	1	1	$\sigma^2 + \sigma^2_{ABC}$

Expected Mean Squares Rules

The EMS for each model term consists of:

- σ^2
- a variance component associated with the term
- a variance component associated with each interaction with the term where all other factors are random

Coefficients:

- for σ^2 is 1
- for all other components is equal to the product of all treatment levels not included in the main effect or interaction

Expected Mean Squares Rules
Example

Model:

Y = A B AB C AC BC ABC

A random, B and C fixed

Expected Mean Squares Rules

Example

A random, B and C fixed

A	
B	
AB	
C	
AC	
BC	
ABC	

Expected Mean Squares Rules

Example

A random, B and C fixed

A	$\sigma^2 + bc\sigma^2_A$
B	$\sigma^2 + c\sigma^2_{AB} + ac\Phi(B)$
AB	$\sigma^2 + c\sigma^2_{AB}$
C	$\sigma^2 + b\sigma^2_{AC} + ab\Phi(C)$
AC	$\sigma^2 + b\sigma^2_{AC}$
BC	$\sigma^2 + \sigma^2_{ABC} + a\Phi(BC)$
ABC	$\sigma^2 + \sigma^2_{ABC}$

Expected Mean Squares
The Controversy

<u>Two approaches</u>:

- Mood (1950)
- Anderson and Bancroft (1952)

The main difference is in how mixed interactions are treated:

- Mood assume mixed interactions to be i.i.d. normal random variables
- Anderson and Bancroft assume that interaction effects sum to zero over levels of the fixed factor

Expected Mean Squares
The Difference

Anderson and Bancroft
Sum-to-Zero Constraint

Source	a F i	b R j	r R k	EMS
A_i	0	b	r	$\sigma^2 + r\sigma^2_{AB} + br\Phi(A)$
B_j	a	1	r	$\sigma^2 + ar\sigma^2_B$
AB_{ij}	0	1	r	$\sigma^2 + r\sigma^2_{AB}$
$\varepsilon_{(ij)k}$	1	1	1	σ^2

Mood
Independence Model

Source	a F i	b R j	r R k	EMS
A_i	0	b	r	$\sigma^2 + r\sigma^2_{AB} + br\Phi(A)$
B_j	a	1	r	$\sigma^2 + r\sigma^2_{AB} + ar\sigma^2_B$
AB_{ij}	1	1	r	$\sigma^2 + r\sigma^2_{AB}$
$\varepsilon_{(ij)k}$	1	1	1	σ^2

Expected Mean Squares
SAS Lab Example

```
The GLM Procedure
Source     Type III Expected Mean Square
field          Var(Error) + 5 Var(field*fert) + 25 Var(field)
fert           Var(Error) + 5 Var(field*fert) + Q(fert)
field*fert     Var(Error) + 5 Var(field*fert)
```

Source	2 R i	5 F j	5 R k	EMS
F_i	1	5	5	$\sigma^2 + 25\sigma^2_F$
F_j	2	0	5	$\sigma^2 + 5\sigma^2_{FF} + 10\phi(F)$
FF_{ij}	1	0	5	$\sigma^2 + 5\sigma^2_{FF}$
$\varepsilon_{(ij)k}$	1	1	1	σ^2

Expected Mean Squares Rules
Independence Model

The EMS for each model term consists of:

- σ^2
- a variance component associated with the term
- a variance component associated with each interaction with the term that contains a random factor (all mixed interactions that include the factor)

Coefficients:

- for σ^2 is 1
- for all other components is equal to the product of all treatment levels not included in the main effect or interaction

Expected Mean Squares Rules
Independence Model Example

A random, B and C fixed

A	
B	
AB	
C	
AC	
BC	
ABC	

Expected Mean Squares Rules
Independence Model Example

A random, B and C fixed

A	$\sigma^2 + \sigma^2_{ABC} + b\sigma^2_{AC} + c\sigma^2_{AB} + bc\sigma^2_{A}$
B	$\sigma^2 + \sigma^2_{ABC} + c\sigma^2_{AB} + ac\Phi(B)$
AB	$\sigma^2 + \sigma^2_{ABC} + c\sigma^2_{AB}$
C	$\sigma^2 + \sigma^2_{ABC} + b\sigma^2_{AC} + ab\Phi(C)$
AC	$\sigma^2 + \sigma^2_{ABC} + b\sigma^2_{AC}$
BC	$\sigma^2 + \sigma^2_{ABC} + a\Phi(BC)$
ABC	$\sigma^2 + \sigma^2_{ABC}$

Comparing Treatment Means
More than One Error Term

<u>Factors</u>:

Fertilizer Rate	4	
Hybrid	9	
Replication	4	eus = 144

Source	4 F i	9 R j	4 R k	EMS	F Test
R_i	0	9	4	$\sigma^2 + 4\sigma^2_{RH} + 36\phi(R)$	MS(R) / MS(RH)
H_j	4	1	4	$\sigma^2 + 16\sigma^2_{H}$	MS(H) / MS(E)
RH_{ij}	0	1	4	$\sigma^2 + 4\sigma^2_{RH}$	MS(RH) / MS(E)
$\varepsilon_{(ij)k}$	1	1	1	σ^2	

Comparing Treatment Means
More than One Error Term

<u>Standard Errors</u>:

Fertilizer Rate $S_{\bar{d}} = \sqrt{\frac{2MS(RH)}{36}}$ df = 24

Hybrid $S_{\bar{d}} = \sqrt{\frac{2MS(E)}{16}}$ df = 108

Rate x Hybrid $S_{\bar{d}} = \sqrt{\frac{2MS(E)}{4}}$ df = 108